Dynamic Force Analysis of Julius Cheezer on Rat Legs

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## **Executive Summary**

A dynamic force analysis of vehicle motion along the connecting track Julius Cheezer revealed a resultant force of 639454 newtons. This resultant force from the vehicles motion causes normal and shear stresses in the A-Frame supports that hold the connecting track. The normal and shear stresses are 1924 MPa and 1.5 MPa respectively while the maximum normal and shear stress of the A36 steel used in the A-Frame are 550 MPa and 58000 MPa respectively.

A factor of safety of 30.15 is generated from the maximum stresses that A-36 Steel can support and the stresses created by the dynamic forces along the track.

This analysis verifies the safety of the A-Frame supports when under a dynamic force of the vehicle on Julius Cheezer.

## **Introduction**

The University of Utah is engineering a brand-new roller coaster with exciting thrills and tracks. This new rollercoaster has been themed after a rat uprising and is designed to support its riders effectively and safely. This coaster consists of three main thrills, one main climb, one vehicle, four connecting tracks, and many supports that frame the whole ride. Julius Cheezer is a section of the track that includes a wide 360-degree banked turn around a 20-meter radius as seen in Figure 1-1.

A close-up of a flexible tube

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| ***Figure 1-1:*** *Aerial view of the Julius Cheezer connecting track. The roller coaster vehicle will approach from the left when looking from the aerial view and go through a 360-degree banked turn around a 20-meter radius.* |

The frame that supports Julius Cheezer is an A-Frame design that consists of tubular legs ranging from 3 meters to 100 meters off the ground as seen in Figure 1-2.

A drawing of a swing set

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| ***Figure 1-2:*** *Front view of the A-Frame supports holding a section of the track. The two pillars come together creating the “A-Frame” and has a section extending down that attaches to the track.* |

The A-Frame connects to Julius Cheezer by holding it between its legs as seen in Figure 1-3. To ensure rider safety around Julius Cheezer, a dynamic force analysis is performed and the resulting forces are compared to the A-Frame’s specifications.

A close-up of a pipe

Description automatically generated

***Figure 1-3:*** *Full Assembly Model of Julius Cheezer with Supporting A-Frames.*

## **Kinematics and Physics of Julius Cheezer**

Riders will enter Julius Cheezer before the 360-degree banked turn at approximately 44.28 meters per second and leave the connecting track at 52.28 meters per second. After entering the 20-meter radius banked turn, the vehicle and its riders will complete the entire wrap-around within 10 seconds. Travelling around the banked turn will result in centripetal force dynamics. The centripetal forces are caused by normal and tangential accelerations as seen in Figure 2-1.

A circle with a line and a line in the middle

Description automatically generated with medium confidence

***Figure 2-1:*** *Breakdown of centripetal acceleration around a curve where R is the radius of the curve, is the acceleration of the body around the curve in the tangential direction, and is the acceleration of the body around the curve in the normal direction.*

The acceleration in the tangential direction is responsible for the vehicle’s motion linearly and allows it to translate forward from the reference point of a passenger. The acceleration in the tangential direction is given by

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

where is the first derivative of velocity, *t* is the time it takes to travel around the curve (seconds), *Vout* is the velocity of the vehicle leaving the connecting track (meters/second), and *Vin* is the velocity of the vehicle entering the connecting track (meters/second). The acceleration in the normal direction is responsible for the translation of the vehicle along the curve and is perpendicular to the tangential acceleration. The acceleration in the normal direction is given by

|  |  |  |
| --- | --- | --- |
|  |  | (2) |
|  |  |  |

where *V* is the velocity of the vehicle around the wrap-around (meters/second), *R* is the radius of the curve (meters), is the first derivative of length, *t* is the time it takes to travel around the curve (seconds), and *l* is the length of the wrap-around (meters).

## **Resulting Forces from Vehicle Motion on Julius Cheezer**

Using Newtons second law, the accelerations caused by the motion around the curved connecting track, Julius Cheezer, can be transformed into forces.

A diagram of a circle with arrows and a red line

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| ***Figure 3-1:*** *Breakdown of centripetal forces around a curve where R is the radius of the curve, is the force of the body around the curve in the tangential direction, is the force of the body around the curve in the normal direction, and F is the resultant force from* . |

The acceleration in the tangential and normal directions became the forces in the tangential and normal directions respectively as seen in Figure 3-1. This was done by utilizing Newtons second law which is given by

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

where *m* is the mass of vehicle with its passengers (kilograms), and *a* is the acceleration of the vehicle around Julius Cheezer (meters/). When combining equations (1) and (2) with equation (3), the force transformation is given by

|  |  |  |
| --- | --- | --- |
|  |  | (4) |
|  |  | (5) |
|  |  |  |

where is the mass of vehicle with the passengers (kilograms), *Vout* is the velocity of the vehicle leaving the connecting track (meters/second), *Vin* is the velocity of the vehicle entering the connecting track (meters/second), *t* is the time it takes to go around the wrap-around, *l* is the length of the wrap-around (meters), and *R* is the radius of the wrap-around. Using the forces calculated in the tangential and normal directions, a resultant force can be found that is indicated in Figure 3-1 by the red vector. The magnitude of the resultant force can be calculated using the equation

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

where is the force in the tangential direction given by equation (4) (Newtons), and is the force in the normal direction given by equation (5) (Newtons).

## **Stresses acting on A-Frame Supports from Dynamic Forces**

The dynamic force analysis of the vehicle along Julius Cheezer creates stresses in the A-Frame supports. Due to the orientation of the forces being horizontal and parallel to the ground, the resultant force, calculated using equation (6), creates normal bending stresses and transverse shear stresses in the A-Frames legs. The maximum transverse shear stress and maximum normal bending stress that result from the resultant force is located at the base of the A-Frame next to ground.

The maximum normal bending stress is given by the equation

|  |  |  |
| --- | --- | --- |
|  |  | (7) |

where *c* is the radius of the A-frame leg (meters), *M* is the maximum bending moment (Newton-meter), and *I* is the moment of inertia of the A-frame leg (. The maximum bending moment can be given by

|  |  |  |
| --- | --- | --- |
|  |  | (8) |

where *F* is the resultant force (Newtons), and *d* is the vertical distance from the ground to where the track connects to the A-frame (meters). The moment of inertia is given by

|  |  |  |
| --- | --- | --- |
|  |  | (9) |

where *d* is the diameter of the A-Frame leg (meters). After substituting equations (8) and (9) into equation (7), the maximum normal bending stress is calculated. The transverse shear stress is given by the equation

|  |  |  |
| --- | --- | --- |
|  |  | (10) |

where *F* is the resultant force (Newtons), and *A* is the cross-sectional area of the cylindrical A-Frame support leg.

## **Results of Force and Stress Analysis**

The accelerations components, force components, and resultant forces shown in Table 1.1 were found using equations 1, 2, 4, 5, and 6. The normal bending stress and transverse shear stress shown in Table 1.2 were found using equations 7 and 10. The material properties of A-36 Steel are shown in Table 1.3 for safety reference.

**Table 1.1:** Dynamic Force Analysis Results

|  |  |  |
| --- | --- | --- |
| Parameter | Symbol | Value |
| Acceleration Tangential |  | 0.8 m/s |
| Acceleration Normal |  | 210.2 m/s |
| Force Tangential |  | 2436 N |
| Force Normal |  | 639450 N |
| Resultant Force | F | 639454 N |

**Table 1.2:** Stress Analysis Results

|  |  |  |
| --- | --- | --- |
| Parameter | Symbol | Value |
| Normal Bending Stress |  | 1924 MPa |
| Transverse Shear Stress |  | 1.5 MPa |

**Table 1.3:** Material Properties of A-Frame A-36 Steel

|  |  |  |
| --- | --- | --- |
| Parameter | Symbol | Value |
| Maximum Normal Stress |  | 550 MPa |
| Transverse Shear Stress |  | 58000 MPa |

## **Conclusion**

A dynamic force analysis of Rat Legs is performed to ensure the safety of University of Utah students along the Ute Coaster’s connecting track Julius Cheezer. As seen in Table 1.1, a dynamic force analysis of the vehicle traveling along Julius Cheezer revealed a resultant force of 639454 Newtons created by the normal and tangential accelerations and the vehicles mass.

Normal bending and transverse stresses were calculated using the resultant force. The stresses experienced by the A-Frames that are generated by the dynamic forces of the vehicle along Julius Cheezer are small in comparison to the maximum stresses of the A36 steel used as the A-Frame legs. As seen in Table 1.2, the normal bending stress and transverse shear stress are 1924 MPa and 1.5 MPa respectively. In Table 1.3, the maximum normal and maximum shear stress that the A36 steel can handle is 550 MPa and 58000 MPa respectively.

This results in a safety factor of 30.14.

In conclusion, the dynamic force analysis of Rat Legs verifies that the A-Frame can adequately and safely support the dynamic forces generated by Julius Cheezer.

**Revisions**

* **Draft 1 to Draft 2**
  + Made the executive summary its own page and broke up its paragraphs to more effectively present information – teacher suggestion
  + Made all image legends no wider than the images – teacher suggestion
  + Removed unnecessary language like “unique” for presenting to peer engineering audiences – teacher suggestion
  + Removed over explanation of topics like angular momentum due to peer audience – teacher suggestion
  + Referenced images after text not before – teacher suggestion
  + Re-organized paragraphs to better show differences between sections – teacher suggestion
  + Introduced tables in the results section with text – teacher suggestion
  + Found errors in calculations and fixed them to match design criteria
  + Broke up conclusion to better present message – teacher suggestion
  + Fixed grammatical and formatting issues in the tables, title page and results – peer suggestion
  + Improved explanation in the results section for better understanding – peer suggestion
* **Draft 2 to Final**
  + Removed and altered sentences that created a false timeline of being past the design stage – teacher suggestion
  + Added an image for further clarification of how the final product looks – teacher suggestion
  + Italicized variable explanations as seen in the equations – teacher suggestion
  + Updated the title page to match current iterations – peer suggestion
  + Changed the tensing of many sentences to match the current timeline of being in the present not the past – peer suggestion.